Name: **Register Number:** **Class:**

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南僑中學

NAN CHIAU HIGH SCHOOL

MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **14 May 2019, Tuesday**

2 hours

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Mdm Chua Seow Ling and Mdm Siak Chock Kwun

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $\sqrt{64^x} = \frac{8^{x+1}}{32^{1-2x}}$, find the value of x. [3]

2 Prove the identity $(\tan x + \sec x)^2 = \frac{1+\sin x}{1-\sin x}$. [2]

3 The equation of a curve is $y = \frac{2x^2}{1-3x}$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Given that x is changing at a constant rate of 0.05 units per second, find the rate of change of y when $x = 3$. [2]

4 Express $\frac{-4x^3+11x^2-16x+9}{x(2x-1)(x^2+3)}$ in partial fractions. [7]

5 **(i)** Express $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2$ in the form $a + b\sqrt{15}$, where a and b are integers. [4]

(ii) Given that $y = 2x^2 - px + 8$ and that $y < 0$ only when $(\sqrt{3} - 1) < x < k$, find the exact value of p and of k. [5]

6 **(i)** Sketch the graph of $y = 2\sqrt{x^3}$ for $x \geq 0$. [1]

(ii) Find the coordinates of the points of intersection of the curve $y = 2\sqrt{x^3}$ and the line $y = -2x + 4$. [5]

7 Solve each of the following equations.

(i) $10^{\log_5 x} = 5$ [3]

(ii) $\log_2(6 - x) - \log_2(x - 2) = 3 - \log_2(2x + 1)$ [4]

8 A curve is such that $\frac{dy}{dx} = 2x^2 - x - 10$. The curve has a maximum y value of 13. Find the equation of the curve. [6]

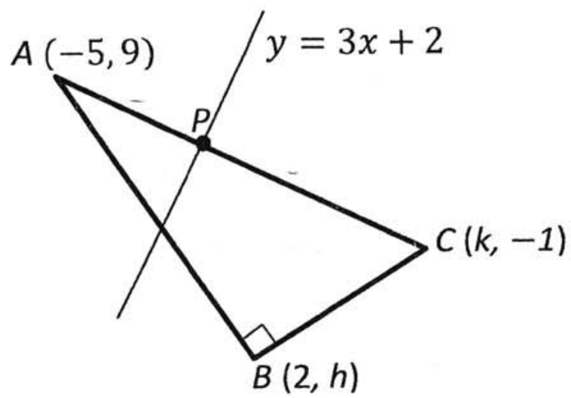
9 It is given that $y = -\frac{1}{9}\ln(3x-2) - 2x + 3$ for $x > \frac{2}{3}$.

(i) Determine, with appropriate workings, whether y is increasing or decreasing. [5]

(ii) Find the range of values of x for which $\frac{dy}{dx}$ is increasing. [2]

10 ***Solutions to this question by accurate drawing will not be accepted.***

The diagram shows a right-angled triangle ABC such that $\angle ABC = 90°$. Given that the coordinates of A, B and C are $(-5, 9)$, $(2, h)$ and $(k, -1)$ respectively where h and k are integers. The line $y = 3x + 2$ meets AC at P such that $5AP = 2AC$.



Find

(i) the coordinates of P, [2]

(ii) the value of k and of h, [4]

(iii) the area of the triangle ABC. [2]

11 **(i)** Find the range of values of x for which $|2x - 3| > 7$. [3]

(ii) Given that the coordinates of the maximum point of the graph $y = a|bx - 3| + c$ is $\left(\frac{3}{4}, 5\right)$, where a, b and c are integers. The y-intercept of the graph is -4.

(a) Find the value of a, of b and of c. [3]

(b) Find the coordinates of the x-intercepts. [4]

12 It is given that $y = 2\cos^2 x - 4\sin^2 x$ for $0 \leq x \leq 2\pi$.

(i) Express y in the form $a + b\cos 2x$, where a and b are integers. [3]

(ii) Hence, state the period and amplitude of y. [2]

(iii) Sketch the graph of $y = 2\cos^2 x - 4\sin^2 x$ for $0 \leq x \leq 2\pi$. [3]

(iv) On the same axes, draw a suitable straight line to find the number of solutions that satisfy the equation $x = 2\pi - 3\pi\cos 2x$ for $0 \leq x \leq 2\pi$. [3]

--- End of Paper ---

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Name: Register Number: Class:

南僑中學

NAN CHIAU HIGH SCHOOL

MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **15 May 2019, Wednesday**

2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Setter: Mrs Sim Hwee Mung and Ms Doris Toh

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integ and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. **(a)** State the values between which each of the following must lie:

   (i) the principal value of $\tan^{-1} x$, [1]

   (ii) the principal value of $\cos^{-1} x$. [1]

   (b) Without using a calculator, find the **exact** value of $\tan 105°$. [3]

2. A curve is such that $\frac{d^2y}{dx^2} = \frac{36}{(1-2x)^3}$. The gradient of the tangent at the point $(-1, 3)$ is $\frac{1}{2}$. Find the equation of the curve. [5]

3. The roots of the equation $x^2 + mx + n = 0$ are α and β, where α β > 0.

Given that $\alpha^2 - \beta^2 = 13$, $\alpha - \beta = -1$ and $2\beta^2 = 72$, find the value of m and of n. [7]

4. Given that $y = (k-2)x^2 - kx + k - x - 2$, find the range of values of k for which y is always positive. [7]

5. An object is heated until it reaches a temperature of T_0 ^{o}C. It is then allowed to cool. Its temperature, T ^{o}C, when it has been cooled for n minutes, is given by the equation $T = 33 + 12e^{-\frac{3}{4}n}$.

(i) Find the value of T_0. [1]

(ii) Find the value of n when $T = 37$ ^{o}C. [1]

(iii) Find the value of n at which the rate of decrease of temperature is 0.67 ^{o}C/minute. [2]

(iv) Explain why the temperature of the object is always greater than 33 ^{o}C. [1]

(v) Sketch the graph of $T = 33 + 12e^{-\frac{3}{4}n}$. [2]

6. The polynomial $f(x) = ax^3 + x^2 + bx + 6$ has a factor of $(x + 2)$ and leaves a remainder of 18 when divided by $(x - 1)$.

(i) Find the value of a and of b. [4]

(ii) Factorise $f(x) = ax^3 + x^2 + bx + 6$ completely. [2]

(iii) **Hence**, using the values of a and b found in (i), solve the equation $a(y-1)^3 + (y-1)^2 + b(y-1) + 6 = 0$. [2]

7. **(i)** Differentiate $sin^3 2x$ with respect to x. [2]

(ii) Hence evaluate the following.

(a) $\int_0^{\frac{\pi}{8}} sin^2 2x cos2x \, dx$ [2]

(b) $\int_0^{\frac{\pi}{8}} cos^3 2x \, dx$ [4]

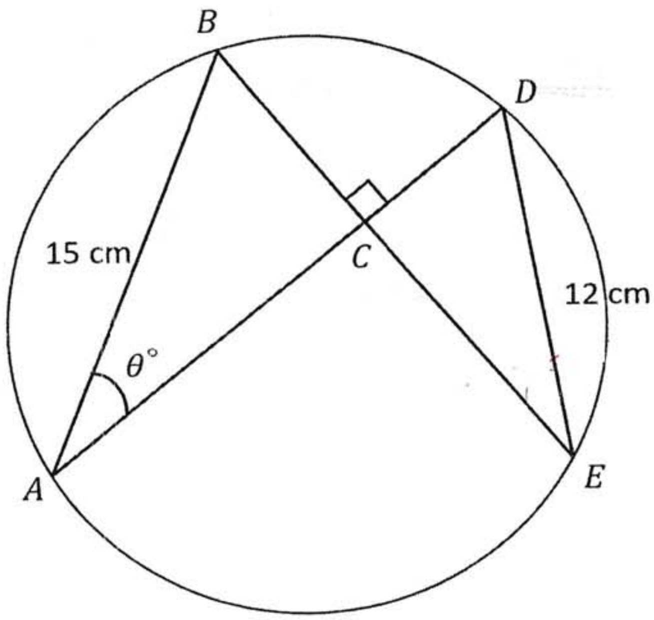
8. In the expansion of $(3 + 5x)^n$, the value obtained when coefficient of x^2 is divided by coefficient of x^3 is 0.3.

(i) Find the value of n. [4]

(ii) Hence, find the term independent of x in the expansion of $(3 + 5x)^n \, (1 - \frac{2}{x})^2$. [5]

9. In the diagram below, BE is perpendicular to AD.

Given that $\angle BAC = \theta°$, where θ is an acute angle, $AB = 15$ cm and $DE = 12$ cm.



(i) Express ~~the~~ AD in the form $R\cos(\theta - \alpha)$, where R is positive and α is an acute angle. [4]

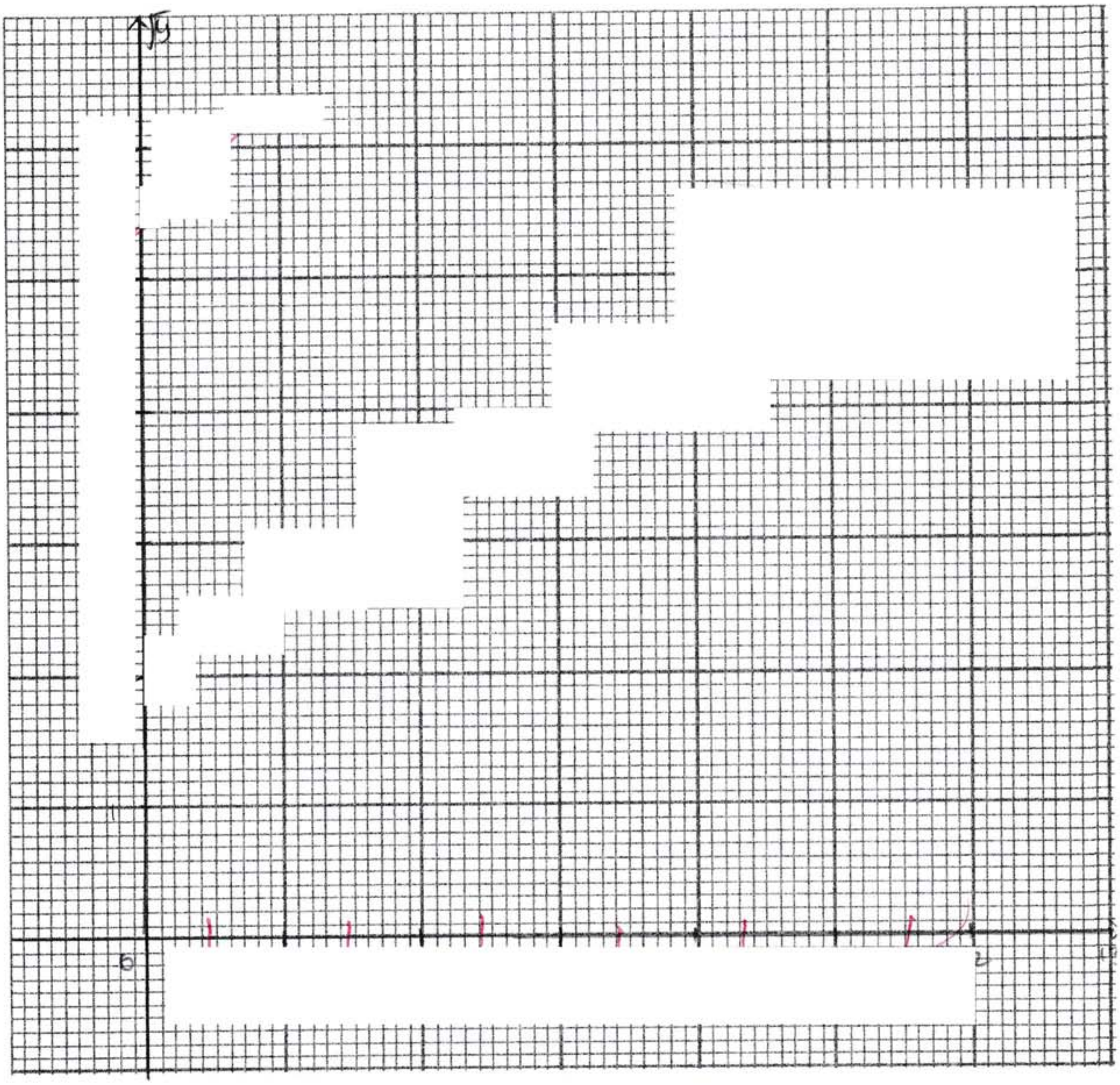
(ii) Find the value of θ for which $AD = 16.5$ cm. [3]

(iii) Given that AD is the diameter, find the length of AD and the corresponding value of θ. [3]

10. The table shows experimental values of two variables x and y. The two variables are related by the equation $b\sqrt{y} = ab + ax^2$, where a and b are non-zero constants. One of the y values have been misprinted.

x	1	1.5	2	2.5	3	3.5
y	5.23	6.98	7.88	14.3	20.9	30.3

(i) Using a scale of 1 cm to 1 unit on the x^2 axis and 2 cm to 1 unit on the $\sqrt{y}$ axis, plot x^2 against $\sqrt{y}$ and draw a straight line graph on the grid provided. [3]



(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Using your graph, identify the abnormal reading and estimate its correct value. [3]

11 **(a)** Find the exact coordinates of the stationary points of the curve $y = 5x^2e^{-3x}$ and determine the nature of the stationary points. [6]

(b) A curve has the equation $y = \frac{x^3+2}{x^2}$. Find the value of k for which the line $y + \frac{27}{23}x = k$ is a normal to the curve. [6]

12. A circle, C_1, has equation $2x^2 - 3x + 2y^2 - \frac{1}{2}(4y - 3) = 0$.

(i) Find the coordinates of the centre and the radius of C_1. [3]

(ii) Show your working clearly whether the point $P\ (-1\,,2)$ lies inside or outside C_1. [2]

(iii) Find the equation of another circle, C_2, which is a reflection of C_1 in the line $y - x - 3 = 0$. [7]

- End of Paper -

NCHS Sec 4 Mid-Year Exam 2019
Additional Mathematics Paper 1

1

$\sqrt{64^x} = \frac{8^{x+1}}{32^{1-2x}}$

$2^{6x} = \left(\frac{2^{3x+3}}{2^{5-10x}}\right)^2$

$2^{6x} = \frac{2^{6x+6}}{2^{10-20x}}$

$6x = 6x + 6 - (10 - 20x)$

$20x = 4$

$x = \frac{1}{5}$ or 0.2

2

$(\tan x + \sec x)^2$

$= \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right)^2$

$= \left(\frac{\sin x + 1}{\cos x}\right)^2$

$= \frac{(\sin x + 1)^2}{\cos^2 x}$

$= \frac{(\sin x + 1)^2}{1 - \sin^2 x}$

$= \frac{(\sin x + 1)^2}{(1 - \sin x)(1 + \sin x)}$

$= \frac{1 + \sin x}{1 - \sin x}$

3i

$y = \frac{2x^2}{1 - 3x}$

$\frac{dy}{dx} = \frac{4x(1-3x) - 2x^2(-3)}{(1-3x)^2}$

$= \frac{4x - 12x^2 + 6x^2}{(1-3x)^2}$

$= \frac{4x - 6x^2}{(1-3x)^2}$

$= \frac{2x(2-3x)}{(1-3x)^2}$

3ii

$\frac{dx}{dt} = 0.05$ unit/s, $x = 3$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= \frac{2(3)(2-9)}{(1-9)^2} \times 0.05$

$= -\frac{21}{640}$ or -0.0328 units/s

4

$\frac{-4x^3 + 11x^2 - 16x + 9}{x(2x-1)(x^2+3)} = \frac{A}{x} + \frac{B}{2x-1} + \frac{Cx+D}{x^2+3}$

$-4x^3 + 11x^2 - 16x + 9$

$= A(2x-1)(x^2+3) + Bx(x^2+3) + x(Cx+D)(2x-1)$

Let $x = 0$, $9 = -3A$

$A = -3$

Let $x = \frac{1}{2}$, $\frac{13}{8}B = \frac{13}{4}$

$B = 2$

Compare x^3, $-4 = 2A + B + 2C$

$C = 0$

Let $x = 1$, $D = 4$

$\therefore \quad \frac{-3}{x} + \frac{2}{2x-1} + \frac{4}{x^2+3}$

5i

$\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2$

$= \frac{5 + 2\sqrt{15} + 3}{5 - 2\sqrt{15} + 3}$

$= \frac{8 + 2\sqrt{15}}{8 - 2\sqrt{15}} \times \frac{8 + 2\sqrt{15}}{8 + 2\sqrt{15}}$

$= \frac{64 + 32\sqrt{15} + 60}{64 - 60}$

$= \frac{124 + 32\sqrt{15}}{4} = 31 + 8\sqrt{15}$

5ii

$2x^2 - px + 8 < 0$

$2[x - (\sqrt{3} - 1)](x - k) < 0$

$2x^2 - 2kx - 2\sqrt{3}x + 2x + 2\sqrt{3}k - 2k < 0$

$2x^2 - (2k + 2\sqrt{3} - 2)x + 2\sqrt{3}k - 2k < 0$

Compare with $2x^2 - px + 8 < 0$,

$2\sqrt{3}k - 2k = 8$

$k(2)(\sqrt{3} - 1) = 8$

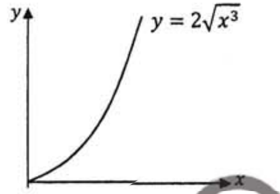
$k = \frac{4}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 2\sqrt{3} + 2$

$p = (2k + 2\sqrt{3} - 2)$

$= 2(2\sqrt{3} + 2) + 2\sqrt{3} - 2$

$= 6\sqrt{3} + 2$

6i



6ii

$2\sqrt{x^3} = -2x + 4$

Square both sides,

$4x^3 = (-2x + 4)^2$

$4x^3 = 4x^2 - 16x + 16$

$4x^3 - 4x^2 + 16x - 16 = 0$

Let $f(x) = 4x^3 - 4x^2 + 16x - 16$

$f(1) = 4 - 4 + 16 - 16 = 0$

$x - 1$ is a factor of $f(x)$

By inspection, $f(x) = (x - 1)(4x^2 + bx + 16)$ where b is a constant.

Compare x^2, $-4 + b = -4$

$b = 0$

$4x^3 - 4x^2 + 16x - 16 = (x - 1)(4x^2 + 16) = 0$

$x - 1 = 0$ or $4x^2 + 16 = 0$

$x = 1$ (rejected)

When $x = 1$, $y = -2 + 4 = 2$

Coordinates = $(1, 2)$

7i

$10^{\log_5 x} = 5$

$\log_5 x = \log_{10} 5$

$x = 5^{\lg 5}$

$= 3.08$

7ii

$\log_2(6 - x) - \log_2(x - 2) = 3 - \log_2(2x + 1)$

$\log_2(6 - x) - \log_2(x - 2) + \log_2(2x + 1) = 3$

$\log_2 \frac{(6-x)}{(x-2)} \times (2x + 1) = 3$

$\frac{(6-x)(2x+1)}{(x-2)} = 2^3$

$12x + 6 - 2x^2 - x = 8(x - 2)$

$2x^2 - 3x - 22 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-22)}}{2(2)}$

$= \frac{3 + \sqrt{185}}{4}$ (4.15) or $\frac{3 - \sqrt{185}}{4}$ (−2.65)(reject)

8

$2x^2 - x - 10 = 0$

$(x + 2)(2x - 5) = 0$

$x = -2$ or 2.5

$\frac{d^2y}{dx^2} = 4x - 1$

$\frac{d^2y}{dx^2} = 4(2.5) - 1 = 9 > 0$, min y when x = 2.5

$\frac{d^2y}{dx^2} = 4(-2) - 1 = -9 < 0$, max y when x = −2

$y = \int 2x^2 - x - 10 \, dx$

$y = \frac{2x^3}{3} - \frac{x^2}{2} - 10x + c$

$13 = \frac{2(-2)^3}{3} - \frac{(-2)^2}{2} - 10(-2) + c$

$c = \frac{1}{3}$

$y = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 10x + \frac{1}{3}$

9i

$\frac{dy}{dx} = \frac{1}{9}\left(\frac{1}{3x-2}\right)(3) - 2$

$= -\frac{1}{3(3x-2)} - 2$

Since $3x - 2 > 0$, $-\frac{1}{3(3x-2)} < 0$,

$\frac{dy}{dx} < 0$ or $-\frac{1}{3(3x-2)} - 2 < 0$

Therefore y is decreasing for $x > \frac{2}{3}$.

9ii

$\frac{d^2y}{dx^2} = -\frac{1}{3}(-1)(3x - 2)^{-2}(3)$

$= \frac{1}{(3x-2)^2}$

$\frac{1}{(3x-2)^2} > 0$

Therefore $x > \frac{2}{3}$.

10i

$y_P = -1 + \frac{10}{5}(3) = 5$

$5 = 3x + 2$

$x = 1$

P (1, 5)

10ii

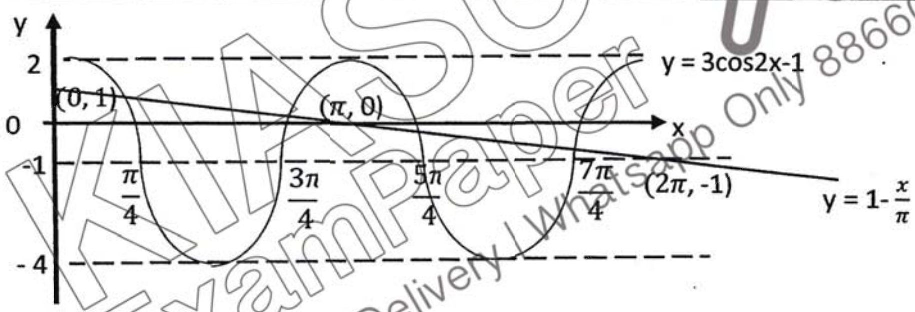
$k = \frac{1 - (-5)}{2}(5) - 5 = 10$

$\left(\frac{9-h}{-5-2}\right)\left(\frac{-1-h}{10-2}\right) = -1$

$(9 - h)(1 + h) = -56$

$h^2 - 8h - 65 = 0$

$h = 13$ (rej) or -5

10iii	$A = \frac{1}{2}\begin{vmatrix} -5 & 2 & 10 & -5 \\ 9 & -5 & -1 & 9 \end{vmatrix}$ $= 70$	11iib	$0 = -3\lvert 4x-3\rvert + 5$ $\lvert 4x-3\rvert = \frac{5}{3}$ $4x - 3 = \frac{5}{3}$ or $4x - 3 = -\frac{5}{3}$ $x = \frac{7}{6}$ or $x = \frac{1}{3}$ $\left(\frac{7}{6}, 0\right)$ and $\left(\frac{1}{3}, 0\right)$
11i	$(2x-3)^2 > 49$ $x^2 - 3x - 10 > 0$ $(x-5)(x+2) > 0$ $x < -2$ or $x > 5$ OR $2x - 3 > 7$ or $2x - 3 < -7$ $x > 5$ or $x < -2$	12i	$y = 2cos^2x - 4(1 - cos^2x)$ $y = 6cos^2x - 4$ $y = 3(cos2x + 1) - 4$ $y = 3cos2x - 1$
11iia	$y = a\lvert 4x-3\rvert + 5$ $-4 = a\lvert -3\rvert + 5$ $a = -3$, $b = 4$, $c = 5$	12ii	Period $= \pi$ Amplitude $= 3$
12iii			
12iv	$y = 3cos2x - 1$ $\frac{x}{\pi} = 2 - 3cos2x$ $3\,cos2x = 2 - \frac{x}{\pi}$ $3\,cos2x - 1 = 1 - \frac{x}{\pi}$ $y = 1 - \frac{x}{\pi}$		

ICHS Sec 4 Mid-Year Exam 2019
Additional Mathematics Paper 2

1ai
$-90^\circ < \tan^{-1}x < 90^\circ$ or
$-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$

1aii
$0^\circ \le \cos^{-1}x \le 180^\circ$ or
$0 \le \cos^{-1}x \le \pi$

1b
$\tan 105^\circ$
$= \tan(60^\circ + 45^\circ)$
$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$
$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$
$= \frac{\sqrt{3}+3+1+\sqrt{3}}{1-3}$
$= \frac{2(\sqrt{3}+2)}{-2}$
$= -\sqrt{3} - 2$

2
$\frac{d^2y}{dx^2} = \frac{36}{(1-2x)^3}$
$\frac{dy}{dx} = \int 36(1-2x)^{-3}\,dx$
$= \frac{36(1-2x)^{-2}}{(-2)(-2)} + c$
$= \frac{9}{(1-2x)^2} + c$
Sub $x = -1, \frac{dy}{dx} = \frac{1}{2}$,
$c = -\frac{1}{2}$

$\frac{dy}{dx} = \frac{9}{(1-2x)^2} - \frac{1}{2}$
$y = \int 9(1-2x)^{-2} - \frac{1}{2}\,dx$
$y = \frac{9}{2(1-2x)} - \frac{1}{2}x + d$
Sub $x = -1, y = 3$,
$3 = \frac{9}{2(3)} + \frac{1}{2} + d$
$d = 1$
$y = \frac{9}{2(1-2x)} - \frac{1}{2}x + 1$

3
$\alpha + \beta = -m$
$\alpha\beta = n$
$(\alpha+\beta)(\alpha-\beta) = \alpha^2 - \beta^2$
$(\alpha+\beta)(-1) = 13$
$\therefore m = 13$

$2\beta^2 = 72$
$\beta = 6$ or -6
Since $\alpha - \beta = -1$,
$\alpha = 5$ (rejected) or $\alpha = -7$
$n = \alpha\beta = (-7)(-6) = 42$

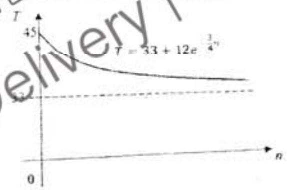
4
$y = (k-2)x^2 - (k+1)x + (k-2)$
$b^2 - 4ac < 0$
$[-(k+1)]^2 - 4(k-2)(k-2) < 0$
$k^2 + 2k + 1 - 4(k^2 - 4k + 4) < 0$
$-3k^2 + 18k - 15 < 0$
$-k^2 + 6k - 5 < 0$
$(k-5)(k-1) > 0$
$k < 1$ or $5 < k$
$k - 2 > 0$
$k > 2$
Answer: $\therefore k > 5$

5i
Sub $n = 0$, $T_0 = 45^\circ$

5ii
$37 = 33 + 12e^{-\frac{3}{4}n}$
$n = 1.46$

5iii
$\frac{dT}{dn} = 12(-\frac{3}{4})e^{-\frac{3}{4}}$
$-0.67 = -9e^{-\frac{3}{4}n}$
$n = 3.46$

5iv
$12e^{-\frac{3}{4}n} > 0$
$33 + 12e^{-\frac{3}{4}n} > 33$
$T > 33^\circ C$ (shown)

5v


6i
$f(-2) = 0$
$-8a - 2b + 10 = 0$
$8a + 2b = 10$
$4a + b = 5$ ------ {1}
$f(1) = 18$
$a + b = 11$ ----- {2}

{1} − {2} :
$3a = -6$
$a = -2$

$-2 + b = 11$
$b = 13$

6ii
$f(x) = -2x^3 + x^2 + 13x + 6$
$= (x+2)(-2x^2 + 5x + 3)$
$= (x+2)(3-x)(2x+1)$

6iii
Let $x = y - 1$
$-(y - 1 + 2)(y - 1 + 3)(2(y - 1) + 1) = 0$
$-(y+1)(y-4)(2y-1) = 0$
$y = -1, 4, 0.5$

7i
$\frac{d}{dx}(\sin^3 2x) = 3\sin^2 2x(2\cos 2x)$
$\frac{d}{dx}(\sin^3 2x) = 6\sin^2 2x\cos 2x$

7iia
$\int_0^{\frac{\pi}{8}} \sin^2 2x\cos 2x\,dx$
$= \frac{1}{6}\int_0^{\frac{\pi}{8}} \frac{d}{dx}(\sin^3 2x)dx$
$= \frac{1}{6}[\sin^3 2x]_0^{\frac{\pi}{8}} = 0.0589$ (3sf)

7iib
$\int_0^{\frac{\pi}{8}} \cos^3 2x\,dx$
$= \int_0^{\frac{\pi}{8}} \cos^2 2x\cos 2x\,dx$
$= \int_0^{\frac{\pi}{8}} (1 - \sin^2 2x)\cos 2x\,dx$
$= \int_0^{\frac{\pi}{8}} \cos 2x\,dx - \frac{1}{6}\int_0^{\frac{\pi}{8}} 6\sin^2 2x\cos 2x\,dx$
$= [\frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x]_0^{\frac{\pi}{8}}$
$= 0.295$ (3sf)

8i
$\binom{n}{2}(3)^{n-2}(5x)^2 = \frac{n(n-1)}{2}3^{n-2}(25x^2)$
$\binom{n}{3}(3)^{n-3}(5x)^3 = \frac{n(n-1)(n-2)}{6}3^{n-3}(125x^3)$
$\frac{\frac{n(n-1)}{2}3^{n-2}(25x^2)}{\frac{n(n-1)(n-2)}{6}3^{n-3}(125x^3)} = \frac{3}{10}$
$\frac{3}{\frac{5(n-2)}{3}} = \frac{3}{10}$
$30 = 5(n-2)$
$6 = n - 2$
$n = 8$

8ii
$(3+5x)^8(1 - \frac{2}{x})^2$
$= (3+5x)^8(1 - \frac{4}{x} + \frac{4}{x^2})$
x term : $\binom{8}{1}(3)^7(5x)$
$= 87480\,x$
x^2 term: $\binom{8}{2}(3)^6(5x)^2$
$= 510300\,x^2$
Term independent of x
$= 3^8(1) - 4(87480) + 4(510300)$
$= 1697841$

9i

angle DEC = θ (angles in same segment)

$15\cos\theta + 12\sin\theta$

$= \sqrt{15^2 + 12^2}\cos(\theta - \tan^{-1}(\frac{12}{15}))$

$= \sqrt{369}\cos(\theta - 38.65981°)$

$= 3\sqrt{41}\cos(\theta - 38.7°)$

$= 19.2\cos(\theta - 38.7°)$

9ii

$16.5 = 3\sqrt{41}\cos(\theta - 38.65981)$

$\cos(\theta - 38.65981) = 0.85896$

Basic angle = 30.80°

$\theta = -30.80°, 69.46028°$

$= 7.9°, 69.5°$

Full mark was given even though students missed out 7.9°.

9iii

$-1 \le \cos(\theta - 38.65981) \le 1$

Max value = $3\sqrt{41}$

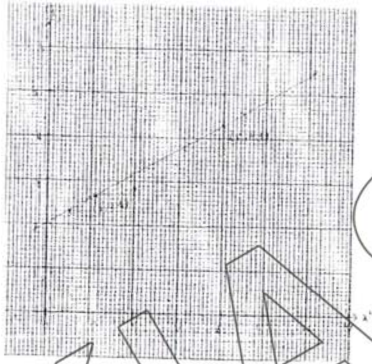
$\cos(\theta - 38.65981) = 1$

$\theta - 38.65981 = 0$

$\theta = 38.7°$

10i

x^2	1	2.25	4	6.25	9	5.50
$\sqrt{y}$	2.29	2.64	2.81	3.78	4.57	12.25



10ii

$b\sqrt{y} = ab + ax^2$,

$\sqrt{y} = \frac{a}{b}x^2 + a$

$m = \frac{4.3-2.6}{8-2} = \frac{17}{60} = 0.28333 = \frac{a}{b}$

Y-intercept = $a = 2$

$\frac{17}{60} = \frac{2}{b}$

b = 7.0588

= 7.06

Accept 6.9 - 7.1

10iii

Abnormal reading when x^2 = 4, $\sqrt{y} = 2.81$

Correct $\sqrt{y}$ should be 3.15, $y = 9.92$ (3.s.f)

Accept $\sqrt{y}$ - 3.05 – 3.15

Accept y = 9.3025 – 10.5625

11a

$y = 5x^2e^{-3x}$ ....(1)

$\frac{dy}{dx} = 10xe^{-3x} + 5x^2e^{-3x}(-3)$

$= 5xe^{-3x}(2 - 3x)$

At stationary point, $\frac{dy}{dx} = 0$

$5xe^{-3x}(2 - 3x) = 0$

$x = 0$, $2 - 3x = 0$ or $e^{-3x} = 0$ (rejected)

$x = 0$, $x = \frac{2}{3}$

Sub $x = 0$ into (1),

$y = 0$

Sub $x = \frac{2}{3}$ into (1),

$y = \frac{20}{9e^2}$

x	-0.1	0	0.1
Sign of $\frac{dy}{dx}$	–	0	+
Sketch of tangent			

(0, 0) is a minimum point

x	0.57	0.67	0.77
Sign of $\frac{dy}{dx}$	+	0	–
Sketch of tangent			

$(\frac{2}{3}, \frac{20}{9e^2})$ is a maximum point

11b

$y = \frac{x^3+2}{x^2}$ ....(1)

$= x + 2x^{-2}$

$\frac{dy}{dx} = 1 - \frac{4}{x^3}$

Equation of normal: $y + \frac{27}{23}x = k$

$y = -\frac{27}{23}x + k$ ....(2)

Gradient of normal = $-\frac{27}{23}$

Gradient of tangent = $\frac{23}{27}$

$\frac{23}{27} = 1 - \frac{4}{x^3}$

$x = 3$

Sub $x = 3$ into (1),

$y = 3\frac{2}{9}$

Sub $(3, 3\frac{2}{9})$ into (2)

$3\frac{2}{9} = -\frac{27}{23}(3) + k$

$k = \frac{1396}{207}$ *or* $6\frac{154}{207}$

12i

$2x^2 - 3x + 2y^2 - \frac{1}{2}(4y - 3) = 0$

$x^2 + y^2 - 1.5x - y + 0.75 = 0$

Centre = $(\frac{-1.5}{-2}, \frac{-1}{-2}) = (0.75, 0.5)$

Radius = $\sqrt{0.75^2 + 0.5^2 - 0.75}$

= 0.25

12ii

Distance of P from centre

$= \sqrt{(0.75 + 1)^2 + (0.5 - 2)^2}$

= 2.3049 > radius

Hence, P lies outside the circle.

12iii

$y = x + 3$

Gradient = 1

Perpendicular gradient = -1

Equation of the line joining the two centres:

$y - 0.5 = -(x - 0.75)$

$y = -x + 1.25$ ...... (1)

$y = x + 3$............ (2)

Sub (2) into (1),

$x + 3 = -x + 1.25$

$x = -\frac{7}{8}$

Sub $x = -\frac{7}{8}$ into (2),

$y = 2\frac{1}{8}$

Let centre of C_2 be (x, y),

$(-\frac{7}{8}, 2\frac{1}{8}) = (\frac{x+0.75}{2}, \frac{y+0.5}{2})$

$x = -2.5$

$y = 3\frac{3}{4}$

Equation of C_2:

$(x + 2.5)^2 + (y - 3\frac{3}{4})^2 = \frac{1}{16}$